Energy-Delay Tradeoff s in a Linear Sequence of Relay Channels

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Abstract

In this paper we aim at characterizing the gain induced by using relay channels in a linear network under both a capacity constraint and a realistic energy model. We express a general model based on a convex optimization problem, allowing us to use numerical tools to obtain similar results for outer and inner bounds to the capacity of the full and half duplex relay channel. We then further the study with more complex networks based on relay channels, especially networks formed by a linear chain of nodes. We describe the Pareto optimal solutions of the minimization problem for with respect to the consumed energy and latency in such a linear network. From the simple case of the linear multi-hop network, we study the gains when implementing a linear chain of relay channels and compare these results to the simpler multi-hop transmission.

1 Introduction

For the past two decades, the exponential growth of mobile communications led to a renewed interest on the capacity of wireless channels and networks, on multiple fronts. When a high number of nodes relaying information in the network are considered, seminal results by Gupta and Kumar gave an asymptotic order of $O(n \log n)$ for the capacity [1]. Many subsequent works aimed to improve on this result, by accounting for mobility [2] or introducing multi-user detection techniques [3]. Computing an optimal resource allocation for large networks, be it time, frequency or power, is also a very hard problem for 2D topologies. Some approaches can reduce the complexity [4] but the size of the search space is in general too large.

To this aim, a common simplification is to consider a unique path in the network, which is then assimilated as a linear network where each node forwards the information from the previous node [5, 6, 7]. This leads to an optimization problem with a single variable distance parameter and a transmission power for each node in the path, a largely simpler framework allowing a deep analytical analysis of a number of problems, like latency or energy minimization. In [6], the authors have proposed to adapt the selection of the relay nodes and their relative transmission powers to optimize power limited or bandwidth limited systems. Capacity and radiated energy are optimized but solutions using a high number of relays are favored since circuit energy is not accounted for. Furthermore, no latency criterion is taken into account. In [7], the authors searched for multi-hop strategies maximizing the total network throughput while minimizing energy consumption. In [8], we focused on a similar problem which we develop further here as the base model for a multihop relay channel.

The second topic at hand is thus the relay channel. Compared to the previous approaches, the relay channel accounts for a direct link between the source and destination in addition to the 2-hop link through the relay node. Its study now spans decades, see [9] for the earliest analysis. A fairly complete review of capacity results for the relay channel and similar node topologies can be found in [10]. Optimizing the capacity of the relay channel under a total power constraint has been the topic of [11], where an algorithm akin to waterfilling is also given for the power allocation. Capacity outage constraints have been studied in asymptotic cases in [12] where the behavior of the total consumed energy is also described. In [13], coherent transmission and reception in the full duplex relay channel case are considered, using cut-set upper bounds and usual lower bounds on the capacity based on decode-and-forward and compress-and-forward protocols at the relay. In his thesis [14], Bae studied the tradeoffs between energy comsumption and transmission delay for half-duplex relay channels for the special case of equal power allocation between the source and relay node.

Our contributions in this paper are as follow:

- In section 3, we state some results for the full and half duplex relay channels. We see that using a relay is not always more efficient than transmitting directly when considering an energy-delay tradeoff. In the second part of this section, we show that under a simple log-distance attenuation model, there's an optimal position for the relay. In the full duplex case, we further show that there's an optimal relay transmission power.

- In section 4, we switch to a linear relay network formed by a succession of relay channels. We develop results akin to [8] for both relay channels, leading to an analytical expression of the per-meter energy/delay Pareto front in each case. With the circuit energy taken into account, we see that the relay channels perform better than the single link, and that the half-duplex relaying model outperforms the full duplex for higher values of the attenuation coefficient.

2 Model description

2.1 Network models

We consider a linear network of nodes transmitting information on a wireless media from a source to a destination. A node i is characterized by its transmission power P_i. Some parameters are common to every nodes, such as a transmission bandwith W and a noise power density N_0. We assume a stable attenuation coefficient $h_{i,j}$ between nodes i and j, with a log-distance propagation model where $h_{i,j} = A_{i,j}.d_{i,j}^{-\alpha}$, $d_{i,j}$ being the distance between the nodes i and j, $A_{i,j}$ a constant with $A_{i,j} > 0$, and α is an environment-dependent coefficient with $\alpha \geq 2$. The transmission and reception powers are normalized with respect to the receiver noise:

$$\gamma_{e(i)} = \frac{P_i}{WN_0} \qquad \gamma_{r(i,j)} = h_{i,j}^2 \frac{P_i}{WN_0} \tag{1}$$

We consider a gaussian model where the 1-hop radio link capacity is defined as:

$$\mathcal{C}\left(\gamma_{r(i,j)}\right) = W \log_2(1 + \gamma_{r(i,j)}) \tag{2}$$

For the remainder of the paper, all log functions are considered as defined in base 2, and the natural logarithm shall be written $\log_e$. For a given information quantity Q, the direct transmission delay is given by $Q/\mathcal{C}\left(\gamma_{r(i,j)}\right)$ on the radio link between i and j. This can be expressed as a **delay per bit**, or latency:

$$D_{i,j} = \frac{1}{\mathcal{C}\left(\gamma_{r(i,j)}\right)} \tag{3}$$

Besides, transmitting Q information bits over the channel between i and j requires an energy equal to $P_i \cdot Q \cdot D_{i,j}$. We can at this point account for additional circuit energy losses E_c on either transmitter or receiver side [5, 8] and normalize the result with respect to Q to obtain an **energy per bit** metric:

$$E_{i,j} = P_i \cdot D_{i,j} + \frac{E_c}{Q} = \frac{\gamma_{e(i)} W N_0}{\mathcal{C}\left(\gamma_{r(i,j)}\right)} + \frac{E_c}{Q} \tag{4}$$

When developing relay channel results, we will consider a simple relay channel under a gaussian hypothesis, as represented on figure 1.

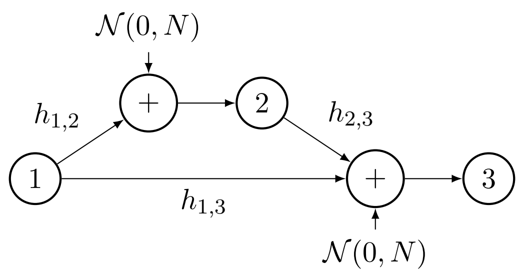

Figure 1: AWGN relay channel

We will consider both the full-duplex (FD) and half-duplex (HD) model. In the full-duplex model, the relay node can receive and transmit at the same time (Fig.1). In the half-duplex case, the relay is listening half the time and transmitting information during the other half, since the simplest half-duplex protocol is to let each node have equal access to the channel. In both models, we consider that the source and relay both transmit their signals towards the destination node, but we do not suppose that they add coherently at the destination.

2.2 Energy considerations

In this section we summarize the results from [8] for single hop channels, or single links (SL). We consider first that $E_c = 0$ in (4). The energy consumption can be expressed as a function of the delay, by inverting (3) to obtain $\gamma_{e(i)}$ as a function of $D_{i,j}$. The per-bit energy (4) can thus be written:

$$E_{i,j} = W N_0 D_{i,j} \frac{2^{1/W D_{i,j}} - 1}{h_{i,j}^2} \tag{5}$$

Because $E_{i,j}$ is a strictly decreasing function of $D_{i,j}$, there's a one to one correspondence between a target latency and the lowest necessary energy to attain it with respect to the gaussian capacity. As shown on Fig.2, this curve delimits the lower end of the achievable region – i.e. the energy-latency trade-offs that can be made in a gaussian single link. Any point on the curve is thus Pareto optimal with respect to energy and latency. This corresponds to the classical Shannon limit. We now consider $E_c \neq 0$, and use the following approximation [8]:

$$E_c = E_0 Q + P_\delta Q D_{i,j} \tag{6}$$

In (6), E_0 stands for a circuit energy related to the processing of 1 bit of data in the network, and P_δ is the additional power consumed by the radiofrequency front-end during the transmission. Normalizing by Q and substituting (6) into (5) yields:

$$E_{i,j} = D_{i,j}\left(WN_0\frac{2^{1/WD_{i,j}}-1}{h_{i,j}} + P_\delta\right) + E_0 \qquad (7)$$

It is seen that E_0 only offsets the tradeoff curve, and as such we will discard it in the remainder of the paper. On the other hand, because of P_δ, $E_{i,j}$ is no longer a strictly decreasing function of $D_{i,j}$. We can see in the next section that it creates a turning point after which the minimum energy required to attain a *higher* latency actually *increases*. This behaviour is actually expected with circuit energy. Low-rate transmissions with high latency will require the radio to be powered on for a longer period of time. If the energy consumption of the radio isn't negligible compared to the transmission power, then most of the dissipation will occur in the circuit without being used to transmit information.

3 Relay transmission optimization

If we want to extend the reasoning of the single link energy/delay tradeoff to the relay channel, we need to extract the optimum power allocation from each relay capacity bounds. Since we consider more than one node and thus more than one power variable in the capacity expressions, it is not straightforward to express it in a usable manner. Such results were derived in specific cases [13, 14]. We chose to work with a generic optimization problem in convex form.

3.1 General formulation of the optimization problem

For the sake of generality, we consider a general network of n nodes in the remainder of this subsection, although the application in this paper is limited to 2 transmitting nodes, the source and the relay. Let $\mathbf{P}^T = (P_1, P_2, \ldots, P_n)$ be the vector of transmission powers, and let $\mathbf{t} = (t_1, t_2, \ldots, t_n)$ be the vector of the fractions of time t_i a node i spends transmitting. We consider the following problem:

$$\begin{array}{rl} \mathit{min.} & \mathbf{t}^T\mathbf{P} \\ \mathit{s.t.} & \mathcal{C}(\mathbf{P}) \geq C^* \\ & \mathbf{P} \succeq 0 \end{array}$$

We define $\succeq$ as the component-wise inequality for vectors. We use this formulation along with lower bounds for the full-duplex relay channel [9] and the half-duplex relay channel [11], in order to obtain Fig.2 using numerical optimization tools.

We can see that all curves for $P_\delta = 0$ have a similar shape, and that considering circuit energy does indeed creates a lowest point beyond which the energy requirements to attain a higher transmission delay increases. The inner and outer bounds are very close in all cases, allowing us to consider only the cutset outer bounds in the remainder of the paper.

Besides, we also consider circuit energy in Fig.2. Indeed, a network with a relay node will consume more energy and may thus perform worse than one without a relay. We consider an energy model where a transmitting node consumes an additional circuit energy $P_{\delta,\mathrm{Tx}}$ and a receiving node $P_{\delta,\mathrm{Rx}}$, with every node in the network sharing the same values. In a simple source-destination network, we can compute $P_\delta^{(\mathrm{SL})} = P_{\delta,\mathrm{Tx}} + P_{\delta,\mathrm{Rx}}$. A full-duplex relay network will have a node listening and transmitting at all time, thus consuming $P_\delta^{(\mathrm{FD})} = 2P_{\delta,\mathrm{Tx}} + 2P_{\delta,\mathrm{Rx}} = 2P_\delta^{(\mathrm{SL})}$. A half-duplex relay network will have, when compared to a single link network, an additional node listening half the time and transmitting during the second half, leading to $P_\delta^{(\mathrm{HD})} = P_{\delta,\mathrm{Tx}} + P_{\delta,\mathrm{Rx}} + \frac{1}{2}(P_{\delta,\mathrm{Tx}} +$

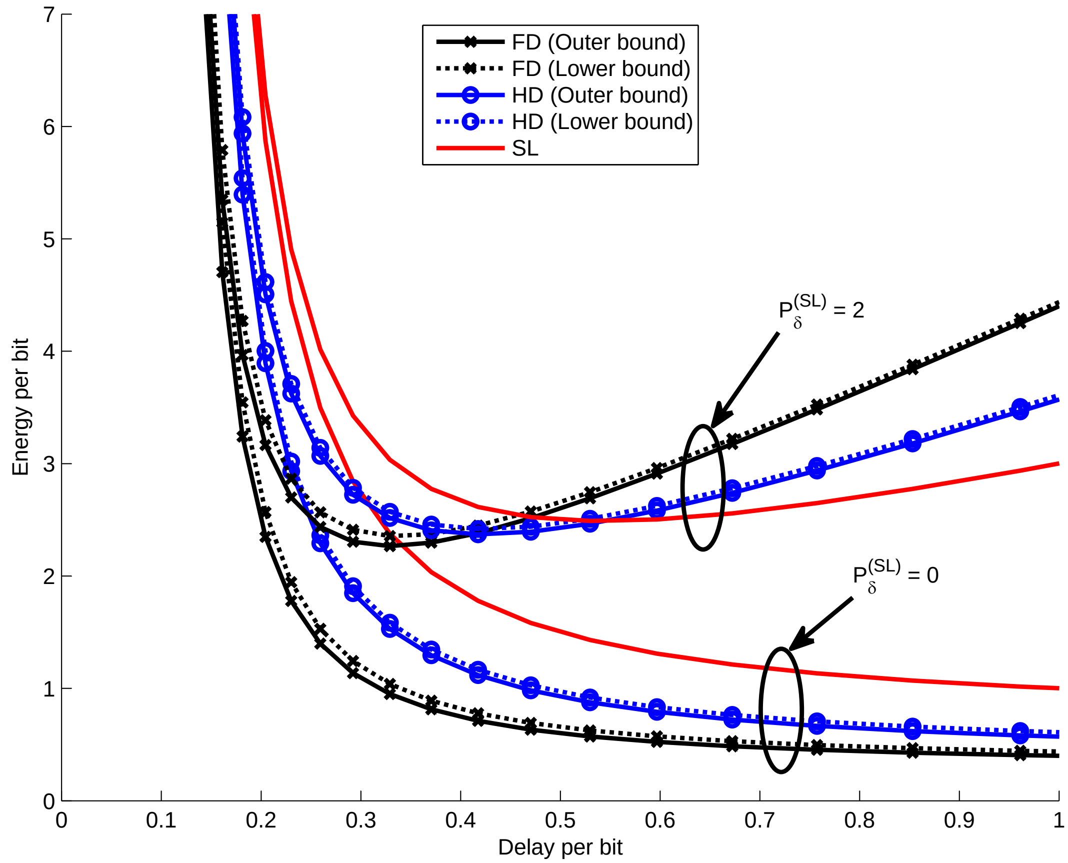

Figure 2: Boundaries of the achievable energy-delay tradeoffs for the single link, the full-duplex relay channel and the half-duplex relay channel. We represent both the cutset outer bounds and the decode and forward inner bounds in the relay cases. For the half-duplex case, the channel separation is 1/2 for each transmission mode. We set $W = N_0 = 1$ and $h_{1,2}/h_{1,3} = h_{2,3}/h_{1,3} = 2$.

$P_{\delta,\mathrm{Rx}}) = \frac{3}{2}P_\delta^{(\mathrm{SL})}$. We can see that the relay channels are no longer unilateraly better than the single link transmission. In fact, for higher values of $P_\delta^{(SL)}$, unless the delay constraint is strong, the single link may be a better choice than the relay channels.

3.2 Log-distance propagation model

We now consider a spatial positioning model, where the source, relay and destination nodes are placed on a line. We introduce the variable d as the relative distance between the source and the relay. We have $h_{1,2}^2 = A_{1,3}d^{-\alpha}d_{1,3}^{-\alpha}$ and $h_{2,3}^2 = A_{1,3}(1-d)^{-\alpha}d_{1,3}^{-\alpha}$. We further write the relay transmission power as a factor of the source transmission power, namely $P_2 = \kappa P_1$. The consumed power in all cases is equal to $\mathbf{t}^T\mathbf{P}$, so that the full-duplex consumes $(1+\kappa)P_1$ and the half-duplex $(1+\frac{1}{2}\kappa)P_1$.

Full-duplex relay. The cutset outer bound for the full-duplex can be written under these premises as [10]:

$$W \log\left(1 + \min\left\{1 + \frac{1}{d^\alpha}\,;\,1 + \frac{\kappa}{(1-d)^\alpha}\right\} \frac{A_{1,3}P_1}{d_{1,3}^\alpha W N_0}\right) \tag{8}$$

The capacity is maximized for $d^{-\alpha} = \kappa(1-d)^{-\alpha}$, such that the optimal $\hat{d} = (1+\kappa^{1/\alpha})^{-1}$. Since we aim to optimize both the energy and delay, we express P_1 as a function of a fixed delay D^* and κ using (3) and the capacity function (8):

$$P_1 = \frac{d_{1,3}^\alpha}{A_{1,3}} W N_0 \left[\frac{2^{1/WD^*} - 1}{1 + (1+\kappa^{1/\alpha})^\alpha}\right] \tag{9}$$

We plug this value into the consumed power $(1+\kappa)P_1$. Taking the derivative w.r.t. κ shows that there exists an optimal $\hat{\kappa}$ for which the Pareto front is lower on both energy and delay accounts. This value verifies:

$$(\hat{\kappa}^{1/\alpha} - \hat{\kappa})(1 + \hat{\kappa}^{1/\alpha})^{\alpha-1} = \hat{\kappa} \tag{10}$$

Half-duplex relay. We now consider a half-duplex relay channel with a time division parameter equal to 1/2. The capacity is written, under the same spatial positioning and propagation model hypotheses [11]:

$$\begin{aligned} &\frac{W}{2} \log\left(1 + \min\left\{1 + \frac{1}{d^\alpha}, 1 + \frac{\kappa}{(1-d)^\alpha}\right\} \frac{A_{1,3}P_1}{d_{1,3}^\alpha W N_0}\right) \\ &\quad + \frac{W}{2} \log\left(1 + \frac{A_{1,3}P_1}{d_{1,3}^\alpha W N_0}\right) \end{aligned} \tag{11}$$

We can see that the capacity will also be maximized for $\hat{d} = (1 + k^{1/\alpha})^{-1}$. For a fixed delay D^* this time we have that P_1 verifies the following relation, with $R_\kappa = 1 + (1+\kappa^{1/\alpha})^\alpha$:

$$2^{2/WD^*} - 1 = \frac{A_{1,3}P_1}{d_h^\alpha W N_0}(1 + R_\kappa + R_\kappa \frac{A_{1,3}P_1}{d_h^\alpha W N_0}) \tag{12}$$

Considering the fact that P_1 is strictly positive, we have only one usable root to (12). Inserting this root in the total consumed power $(1 + \frac{1}{2}\kappa)P_1$, it is possible to show that the derivative of the resulting expression will cancel on a value that is dependent on the delay D^*. We can thus say that there is no universally *best* value for κ in the half-duplex case. We can nevertheless give a specific value of κ optimized if we consider the source-destination SNR to be high enough to write $\log(1 + x) \approx \log(x)$ in (11), leading to:

$$P_1 = \frac{d_{1,3}^\alpha}{A_{1,3}} W N_0 \left[\frac{2^{1/WD^*}}{\sqrt{1 + (1+\kappa^{1/\alpha})^\alpha}}\right] \tag{13}$$

An optimal $\hat{\kappa}$ would thus verify, after some manipulations:

$$(\hat{\kappa}^{1/\alpha}(1 - \frac{1}{2}\hat{\kappa}) - \hat{\kappa})(1 + \hat{\kappa}^{1/\alpha})^{\alpha-1} = \hat{\kappa} \tag{14}$$

Both (10) and (14) don't yield closed-form expressions for $\hat{\kappa}$, but we know that they are unique and they can be solved numerically by fixed point methods – and are in fact expressed as such.

Furthermore, we only have to compute them once for each α we choose in numerical applications, and this parameter is usually fixed for a simulation environment. The same goes for the value of $\hat{d}$ which only depends on α and κ in both cases. We note the values of $\hat{d}$ and $\hat{\kappa}$ for different values of α in Tab.1.

The harsher the attenuation is, the closer to the equidistant position the relay gets, and the higher its transmission power is. This is intuitively satisfying ; when the attenuation is weak, the relay gives a performance boost but the direct path still has a strong contribution, whereas under stronger attenuation the role of the relay is increasingly important in transmitting the information since the direct path gets weaker.

4 Linear network optimization

The previous section dealt with the end-to-end capacity of single-hop and relay channels, using for the latter well-known bounds on its capacity. We extend in this section these results to the case of a linear multi-hop transmission. In previous work, we studied the basic problem, considering a packet error metric for the links first and then the capacity – see [8] and references therein. Our analysis starts as in [7], where the authors further minimize the energy consumption without taking the latency into account, only for specific scenarios. We do a brief presentation of the results of [8] in section 4.1 where each hop is a single link, and we then extend the results using relay channels in the subsequent sections.

4.1 Single link case

We consider that some information quantity Q has to be transmitted over a very long distance, where we are completely free to choose the number, location and transmission power of relay nodes along the path. If the environment is homogeneous and if the attenuation is only a function of the distance between each pair of nodes, then a regular positioning of each relay along the line segment joining the sender and receiver is optimal [6]. The optimization is thus limited to finding the optimal couple (d_h, γ_e) where d_h is the internode distance and γ_e is the transmission power of each node normalized over WN_0.

As proposed in [8], we divide our metrics from (4) and (3) by the distance to obtain per-bit-per-meter energy, and per-bit-per-meter delay:

$$\delta E(d_h, \gamma_e) = \frac{E}{d_h} = \frac{(WN_0\gamma_e + P_\delta)}{d_h \mathcal{C}(\gamma_r)} \tag{15}$$

	Full-duplex		Half-duplex	
α	$\hat{\kappa}$	$\hat{d}$	$\hat{\kappa}$	$\hat{d}$
2	0.38	0.62	0.27	0.66
2.5	0.56	0.56	0.38	0.63
3	0.69	0.53	0.45	0.61
3.5	0.78	0.52	0.50	0.59
4	0.85	0.51	0.54	0.58

Table 1: Optimal values for $\hat{d}$ and $\hat{\kappa}$ for both the half-duplex (HD) and full-duplex (FD) relay channels.

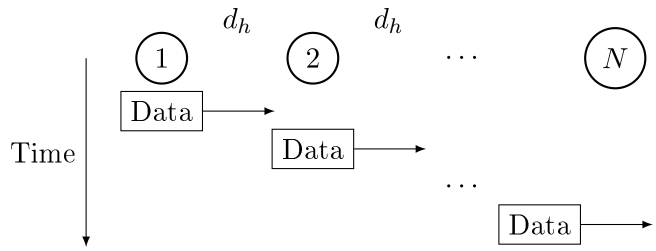

Figure 3: Linear network scenario for a succession of single link channels

$$\delta D(d_h, \gamma_e) = \frac{D}{d_h} = \frac{1}{d_h \mathcal{C}\left(\gamma_r\right)} \tag{16}$$

Using the relation between γ_e, γ_r and d_h we can write (15) and (16) as :

$$\delta E(\gamma_r, \gamma_e) = \frac{(WN_0\gamma_e + P_\delta)}{(A_0\gamma_e)^{1/\alpha}} \cdot \frac{\gamma_r^{1/\alpha}}{\mathcal{C}\left(\gamma_r\right)} \tag{17}$$

$$\delta D(\gamma_r, \gamma_e) = \frac{1}{(A_0\gamma_e)^{1/\alpha}} \cdot \frac{\gamma_r^{1/\alpha}}{\mathcal{C}\left(\gamma_r\right)} \tag{18}$$

These turn out to be separable in γ_e and γ_r. We can write the following proposition:

Proposition 1 ([8]). *In an homogeneous environment characterized by a power law attenuation* (1)*, a linear network with transmitter and receiver power densities* $(\hat{\gamma}_e, \hat{\gamma}_r)$ *is Pareto optimal w.r.t. energy and latency if and only if:*

$$\hat{\gamma}_r = \arg\min \frac{\gamma_r^{1/\alpha}}{\mathcal{C}\left(\gamma_r\right)} \tag{19}$$

$$\hat{\gamma}_e \geq \frac{P_\delta}{WN_0(\alpha - 1)} \tag{20}$$

Proof. The cost functions are separable in γ_r and γ_e, and these variables can vary independently. We can minimize both (17) and (18) with respect to γ_r with the condition (19). We have (18) strictly decreasing with respect to γ_e, but not (17). Basic calculus shows that the derivative of (17) w.r.t. γ_e cancels once for γ_e verifying (20), and is strictly positive beyond this value. The Pareto-optimal point with the lowest energy requirement is thus computed with γ_e verifying (20) with equality, while greater values of γ_e will achieve a lower delay at a higher energy cost, thus spanning the Pareto front.

4.2 Relay channel

In the previous section, we varied the distance and source node power to obtain the Pareto front. In the relay case, as represented on Fig.4, we have *a priori* more degrees of freedom in our optimization, namely the relay node transmission power, and the relative relay position. We know that an optimal relative relay position $\hat{d}$ exists in duplex cases from section 3.2. This reduces the degrees of freedom of the problem by one variable. We use the previous notation $R_\kappa = 1 + (1 + \kappa^{1/\alpha})^\alpha$. Like the single link case, we use the variable changes $\gamma'_e = \frac{P_1}{WN_0}$ and $\gamma'_r = A_{1,3}R_\kappa d_h^{-\alpha}\gamma'_e$. We can write:

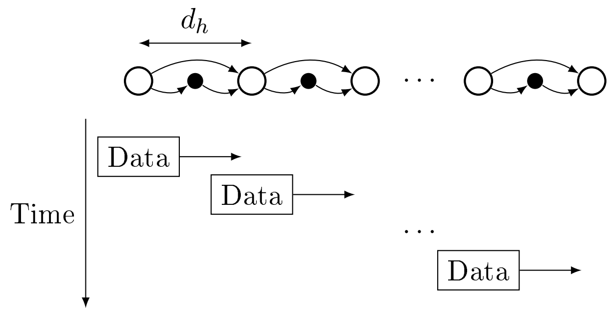

Figure 4: A chain of relay channels.

$$\delta E(\gamma_r', \gamma_e') = \frac{(WN_0(1+\kappa)\gamma_e' + P_\delta)}{(A_{1,3}R_\kappa \gamma_e')^{1/\alpha}} \cdot \frac{\gamma_r'^{1/\alpha}}{\mathcal{C}\left(\gamma_r'\right)} \tag{21}$$

$$\delta D(\gamma_r', \gamma_e') = \frac{1}{(A_{1,3}R_\kappa \gamma_e')^{1/\alpha}} \cdot \frac{\gamma_r'^{1/\alpha}}{\mathcal{C}\left(\gamma_r'\right)} \tag{22}$$

We have the same function of κ as in (9) we can deduce that for the full-duplex relay channel there exists an optimal $\hat{\kappa}$ minimizing the consumed energy for a fixed delay, which depends only on the attenuation coefficient. We can thus enunciate the following proposition:

Proposition 2. *We consider an homogeneous environment characterized by a power law attenuation* (1)*. A linear full duplex relay network with equivalent transmitter and receiver power densities* $(\hat{\gamma}_e', \hat{\gamma}_r')$ *is Pareto optimal w.r.t. energy and latency if and only if:*

$$\hat{\kappa} = (\hat{\kappa}^{1/\alpha} - \hat{\kappa})(1 + \hat{\kappa}^{1/\alpha})^{\alpha-1} \tag{23}$$

$$\hat{\gamma}_r' = \arg\min \frac{\gamma_r'^{1/\alpha}}{\mathcal{C}\left(\gamma_r'\right)} \tag{24}$$

$$\hat{\gamma}_e' \geq \frac{P_\delta}{WN_0(\alpha-1)(\hat{\kappa}+1)} \tag{25}$$

Proof. The proof follows along the lines of Proposition 1 for (24) and (25). The condition (23) comes from section 3.2.

We now consider the half-duplex relay channel under the same hypotheses and a channel access time division $t = 1/2$. Using $\gamma_e' = \frac{P_1}{WN_0}$ and $\gamma_r' = A_{1,3} d_h^{-\alpha} \gamma_e'$, we can write the optimization goals in their separable form:

$$\delta E(\gamma_r', \gamma_e') = \\ \frac{(WN_0(1+\frac{1}{2}\kappa)\gamma_e' + P_\delta)}{(A_{1,3}\gamma_e')^{1/\alpha}} \cdot \frac{2\gamma_r'^{1/\alpha}}{\mathcal{C}\left(\gamma_r'(1+R_\kappa+R_\kappa\gamma_r')\right)} \tag{26}$$

$$\delta D(\gamma_r', \gamma_e') = \frac{1}{(A_{1,3}\gamma_e')^{1/\alpha}} \frac{2\gamma_r'^{1/\alpha}}{\mathcal{C}\left(\gamma_r'(1+R_\kappa+R_\kappa\gamma_r')\right)} \tag{27}$$

The form of the metrics has changed slightly. The capacity function contains a quadratic function of the variable γ'_r, meaning R_κ and thus κ now affects both part of the metrics, which will lead to a more complex form for the Pareto front. Like in section 3.2, we may consider simplifications as before and thus use the value of $\hat{\kappa}$ as computed in section 3.2. We have the following proposition:

Proposition 3. *We consider an homogeneous environment characterized by a power law attenuation (1). A linear half duplex relay network with equivalent transmitter and receiver power densities $(\hat{\gamma}'_e, \hat{\gamma}'_r)$, where the relay uses an arbitrary fixed fraction κ of the source transmission power density, is Pareto optimal w.r.t. energy and latency if and only if:*

$$\hat{\gamma}'_r = \arg\min \frac{\gamma_r'^{1/\alpha}}{\mathcal{C}\left(\gamma'_r(1 + R_\kappa + R_\kappa \gamma'_r)\right)} \tag{28}$$

$$\hat{\gamma}'_e \geq \frac{P_\delta}{W N_0 (\alpha - 1)(\frac{1}{2}\kappa + 1)} \tag{29}$$

Proof. The proof follows along the lines of Proposition 1 and Proposition 2.

5 Applications

We can apply these results to obtain a comparison between our models. For each solution set, we have to solve an equation in the form of (19), which takes the form of a fixed point:

$$\alpha \hat{\gamma}_r \log_e(2) = (\hat{\gamma}_r + 1) \log_e(\hat{\gamma}_r + 1) \tag{30}$$

We use a variable change $x = \hat{\gamma}_r + 1$ and after some manipulations, we have that the solution for x is in the form of the Lambert W function. Doing the inverse variable change leads to:

$$\hat{\gamma}_r = \frac{W_{-1}\left(2^{-\alpha}(1 - \alpha \log_e(2))\right)}{1 - \alpha \log_e(2)} - 1 \tag{31}$$

Because we require that $\hat{\gamma}_r \geq 0$ for any value of $\alpha \geq 2$, a condition that will not be met by the principal branch $W_0(\cdot)$, we restrict ourselves to the branch $W_{-1}(\cdot)$.

For the half-duplex case, we can deduce from (28) that a critical point will verify the following relation, although since no obvious variable change leads to a closed-form solution we resort to a numerical fixed point algorithm to solve it:

$$\frac{\mathcal{C}\left(\hat{\gamma}'_r(1 + R_\kappa + R_\kappa \hat{\gamma}'_r)\right)}{\mathcal{C}'\left(\hat{\gamma}'_r(1 + R_\kappa + R_\kappa \hat{\gamma}'_r)\right)} = \alpha \hat{\gamma}'_r (1 + R_\kappa + 2R_\kappa \hat{\gamma}'_r) \tag{32}$$

We consider a set of values from [15], where $P_\delta^{(SL)} = 120\,\text{mW}$, a frequency bandwith of 1MHz, and a cumulated noise figure of 20 dB. We obtain the figure 5 for different values of α. We can see that the highest the value of the attenuation coefficient, the best the half-duplex relay network performs. Furthermore, it is more interesting in every case on a pure energy/delay tradeoff consideration to use a relay channel.

Considering numerical results on the optimal transmission power and node distance in Tab.2, we see that the required transmission power is quite high. Indeed, the nodes have to compensate for circuit energy and use a similar value for their transmission power in order to stay efficient. This in turn increases the inter-node distance because the received SNR constraint is relatively low. This leads to the conclusion that for dense networks, as alpha increases, half-duplex relays will be the best choice from an energy-delay tradeoff standpoint.

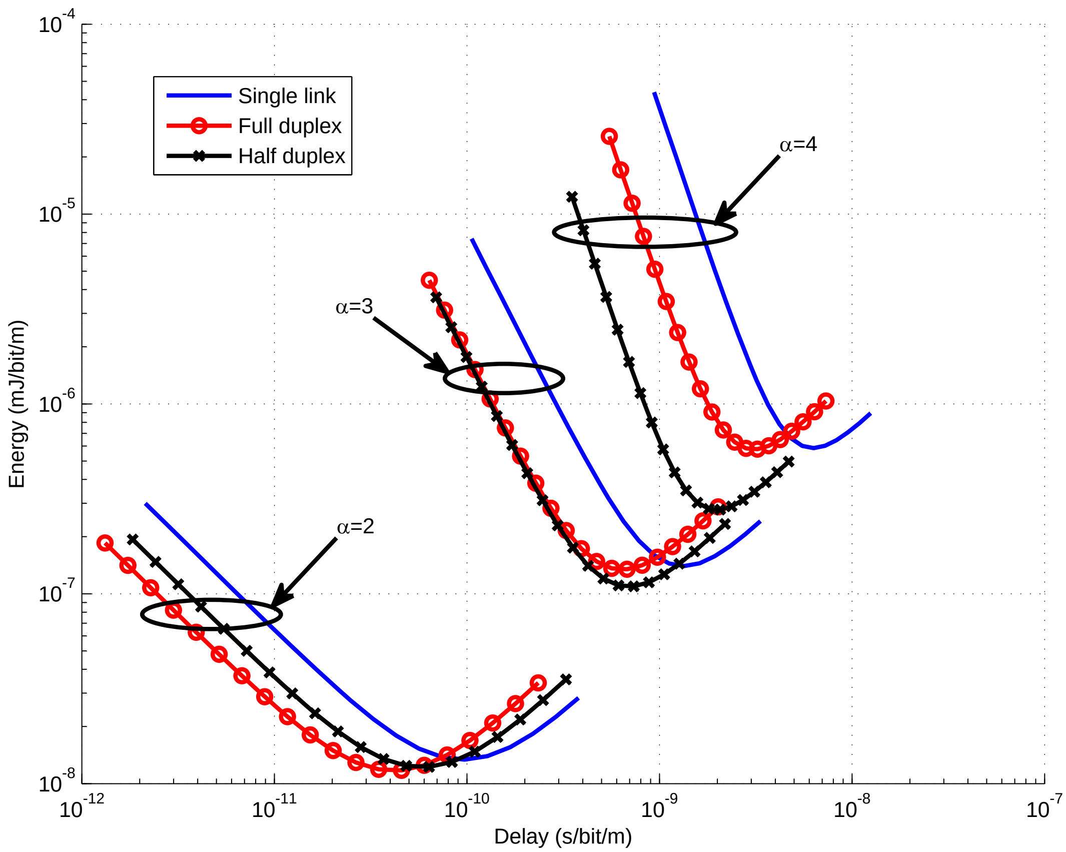

Figure 5: Front of the energy-delay tradeoff using realistic values for the single link and the relay channels.

6 Conclusions and perspectives

In this paper, we studied relay channels under both criteria of delay and required energy, with a realistic energy consumption model. We then studied linear networks where a chain of nodes relay some information from a source to a destination. We derived comparisons between single links and relay channels in such a situation. Both half-duplex and full-duplex relay channels perform better than the single link, the half-duplex relay channel becomes more efficient in our framework than the full-duplex relay channel as the attenuation coefficient increases. A key conclusion for routing protocols in dense networks is that a half-duplex model will allow for better energy-delay tradeoffs, as long as the network is dense enough, since it requires the nodes to be closer to each other.

The perspectives of this work are numerous. We plan to study more the effects and benefits of such metrics in the design of actual network protocols. We could further consider refined energy models where the consumption of different circuits and their relation to the transmission duration or transmission power are more precise. The work we present here is also limited to flat gaussian channels, and we plan to extend it to fading channels and more complex relay schemes.

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	Single link		Full-duplex		Half-duplex	
α	P_1	d_h	P_1	d_h	P_1	d_h
2	20.8	839.9	22.4	1921.9	22.0	2047.4
3	17.8	115.5	18.5	241.1	18.7	87.4
4	16.0	38.1	16.4	77.4	16.7	20.8

Table 2: Coordinates for the Pareto optimal point with the lowest total energy value under our energy/delay model. Powers are in dBm and distances in meters.

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